APPROXIMATE SOLUTION OF THE EQUATIONS OF HEAT-EXCHANGER DYNAMICS ON A DIGITAL COMPUTER

I. P. Simakov and A. E. Solov'ev

UDC 536.27

A modification of the method of lines is proposed for the approximate solution of the system of partial differential equations describing the thermal dynamics of a multistage forced-circulation heat exchanger (HE).

A heat exchanger is, in a mathematical sense, one of the most involved of the dynamical links comprising a steam-energy installation. An adequate mathematical description of a heat exchanger involves the study and solution of a nonlinear boundary-value problem for partial differential equations.

The heat exchanger design (Fig. 1) for which calculations were made is a "tube-in-a-tube" type, thin-walled and thermally insulated from the external medium.

The economizer–evaporator section and the superheated steam section are divided by a separator; the water is taken from the separator by a pump and fed into the economizer portion of the HE. At the economizer portion the water undergoes heating to the boiling temperature $t_s(p)$ and, farther along, there is generation of saturated vapor at the evaporator portion. The boundary for onset of boiling of the water moves with a change in the work mode of the HE.

In deriving the mathematical equations of the HE we make the following conventional assumptions: there is no heat transfer with the environment; the temperature and speed of the heat-transfer agent is the same at all points of a cross section; heat conduction in the direction of motion of the heat-transfer agent is negligibly small in comparison with convective heat transfer; a lumped-parameter description of hydrodynamic processes may be assumed since the heat-transfer agent and the water are incompressible media; heat transfer between the heat-transfer agent and the water in the HE takes place only through the tube walls; values of the densities, specific heat capacities, and thermal-conductivity coefficients are taken to be mean integral values over the temperature range of variation considered; the thermal state of the heat-transmitting tubes through the tube thickness at a given section may, with sufficient accuracy, be characterized by a single (mean) temperature.

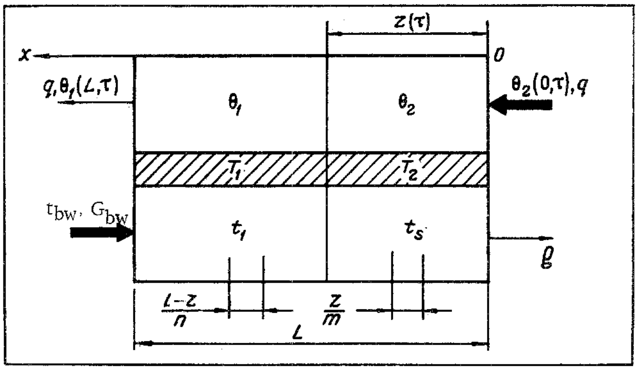

Fig. 1. Basic HE design arrangement.

With these assumptions we may describe the nonstationary working regimes of the HE, when the distribution of the parameters is taken into account, by means of a system of nonlinear partial differential equations [2]:

evaporator section: $0 \le x \le z(\tau)$

$$(c\gamma s)_a \frac{\partial \theta_2}{\partial \tau} = -c_a G_n \frac{\partial \theta_2}{\partial x} - k_1(q)\,\Pi\,(\theta_2 - T_2); \tag{1}$$

$$(c\gamma\delta)_W \frac{\partial T_2}{\partial \tau} = k_1(q)(\theta_2 - T_2) - k_2(p,\ q)(T_2 - t_s); \tag{2}$$

$$t_s = t_s[p(\tau)]; \tag{3}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 6, pp. 1053-1059, December, 1971. Original article submitted January 8, 1971.

© 1974 Consultants Bureau, a division of Plenum Publishing Corporation, 227 West 17th Street, New York, N. Y. 10011. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission of the publisher. A copy of this article is available from the publisher for $15.00.

economizer section: $z(\tau) \leq x \leq L$

$$(c\gamma s)_{a} \frac{\partial \theta_1}{\partial \tau} = -c_a G_n q \frac{\partial \theta_1}{\partial x} - k_1(q)\Pi(\theta_1 - T_1); \tag{4}$$

$$(c\gamma\delta)_W \frac{\partial T_1}{\partial \tau} = k_1(q)(\theta_1 - T_1) - k_2(G_W)(T_1 - t_1); \tag{5}$$

$$(c\gamma s)_W \frac{\partial t_1}{\partial \tau} = c_W G_W \frac{\partial t_1}{\partial x} + k_2(G_W)\Pi(T_1 - t_1), \tag{6}$$

with the corresponding boundary conditions

$$\theta_2(0, \tau) = \theta_2(\tau),\ \theta_1(z, \tau) = \theta_2(z, \tau),\ t_1(L, \tau) = t_{bw}(\tau), \quad t_1(z, \tau) = t_s(p) \tag{7}$$

and initial conditions

$$\theta_i(0, x) = \theta_i^0(x),\ T_i(0, x) = T_i^0(x),\ t_1(0, x) = t_1^0(x), \quad t_s(p) = t_s^0(p) \quad (i = 1, 2). \tag{8}$$

We consider the temperature distribution functions θ, T, and t (temperatures of the alloy, the wall, and the steam–water mixture) to be continuous functions with piecewise-continuous derivatives through the first-order inclusive:

$$\theta_i = \theta_i(x, \tau),\ T_i = T_i(x, \tau),\ t_1 = t_1(x, \tau),\ t_s = t_s[p(\tau)] \quad (i = 1, 2),$$

excluding the point $x = z(\tau)$ at which $T_i(z, \tau)$ $(i = 1, 2)$ has finite discontinuities of the first order.

The functions $\theta_2(\tau)$ and $t_{bw}(\tau)$ are assumed to be continuous.

The difficulty of solving this system of partial differential equations (1)-(6) arises in that the boundary $z(\tau)$, where boiling of the water commences, varies with the time, this variation being defined implicitly by the initial system of equations and by the system of initial conditions (8) and the boundary conditions (7).

We obtain the equation for the time-varying boundary of the evaporator section from Eq. (6), which we consider at the point $x = z_{+0}(\tau)$. Noting that $t_1 = t_1[z(\tau), \tau]$, we write down the expression for the total differential:

$$dt_1(z_{+0}, \tau) = \left.\frac{\partial t_1}{\partial x}\right|_{x=z_{+0}(\tau)} \frac{dz}{d\tau} d\tau + \left.\frac{\partial t_1}{\partial \tau}\right|_{x=z_{+0}(\tau)} d\tau. \tag{9}$$

The notation $z_{+0}(\tau)$ (subscript +0) signifies that we are considering the derivative at the point $x = z(\tau)$, the evaluation being made from the side of the economizer zone. This needs to be made more precise since the partial derivative with respect to the water temperature undergoes a discontinuity at the point $x = z(\tau)$.

Substituting the expression $\partial t_1/\partial\tau|_{x=z_{+0}(\tau)}$ from Eq. (9) into Eq. (6), we obtain

$$\left.\frac{dt_1}{d\tau}\right|_{x=z_{+0}(\tau)} = \left.\frac{\partial t_1}{\partial x}\right|_{x=z_{+0}(\tau)} \left(w_{II} + \frac{dz}{d\tau}\right) + \frac{k_2(G_W)\Pi}{(c\gamma s)_W}(T_1 - t_1)|_{x=z_{+0}(\tau)}. \tag{10}$$

At the boundary of the economizer and evaporator portions

$$t_1[z(\tau), \tau] = t_s[p(\tau)]. \tag{11}$$

From the expression (11) it follows that

$$\frac{dt_1}{d\tau} = \frac{\partial t_s}{\partial p} \cdot \frac{dp}{d\tau}. \tag{12}$$

Substituting Eq. (12) into Eq. (10), we obtain an equation for the variable boundary for the onset of boiling:

$$\frac{dz}{d\tau} = \frac{\dfrac{\partial t_s}{\partial p} \cdot \dfrac{dp}{d\tau} - \dfrac{k_2(G_W)\Pi}{(c\gamma s)_W}(T_1 - t_1)|_{x=z}}{\left.\dfrac{\partial t_1}{\partial x}\right|_{x=z_{+0}}} - w_{II}. \tag{13}$$

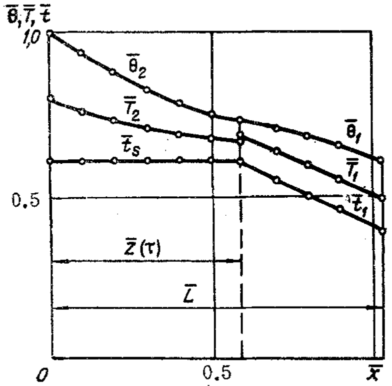

Fig. 2. Stationary distribution of temperatures in the HE.

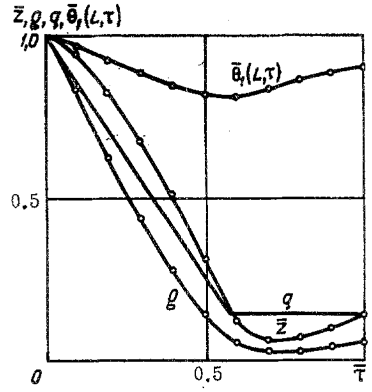

Fig. 3. Results of HE calculations for linear perturbations.

Following the method of lines [1] we subdivide the economizer and evaporator portions into n and m equal parts, respectively, the lengths of which are functions of the time:

$$x_i(\tau) = z + i\frac{L-z}{n} \quad (i = 0, \ldots, n), \tag{14}$$

$$x_i(\tau) = i\frac{z}{m} \quad (i = 0, \ldots, m). \tag{15}$$

Henceforth we consider the temperatures of the heat-transfer agent and the wall at "moving" points on the economizer (Eq. (14)) and the evaporator (Eq. (15)) portions. Employing the method of subdivision of the lines (14), we represent the derivative at the point $x = z_{+0}(\tau)$ by the difference relation

$$\left.\frac{\partial t_1}{\partial x}\right|_{x=z_{+0}(\tau)} \simeq \frac{t_1\left(z+\frac{L-z}{n},\ \tau\right) - t_1(z,\tau)}{\frac{L-z}{n}} = \frac{t_1(x_1,\ \tau) - t_s}{\frac{L-z}{n}}. \tag{16}$$

In this case we can write a differential equation connecting the evaporator zone length with the temperatures:

$$\frac{dz}{d\tau} = \frac{\frac{\partial t_s}{\partial p}\cdot\frac{dp}{d\tau} - \frac{k_2(G_W)\Pi}{(c\gamma s)_W}\left[T_1(z,\ \tau) - t_s\right]}{n\left[t_1(x_1,\ \tau) - t_s\right]}(L-z) - w_{11}. \tag{17}$$

We now find differential equations the solutions of which determine the temperatures of the heat-transfer agent and the wall at the "moving" points of the economizer and evaporator portions.

Since all the temperatures of the economizer and evaporator portions (T_i, θ_i, t_1, t_s (i = 1, 2)), conditionally denoted from now on by $u[x(\tau), \tau]$ for brevity, are functions of the coordinate and the time, we may write the expression for the total differential as

$$du\left[x_i(\tau)\right] = \frac{\partial u(x_i)}{\partial x}\cdot\frac{dx_i}{d\tau}d\tau + \frac{\partial u(x_i)}{\partial \tau}d\tau \quad (i = 0, \ldots, n). \tag{18}$$

Substituting into Eq. (18) the value of $dx_i/d\tau$ from Eq. (14) for the economizer portion and the values from Eq. (15) for the evaporator portion, we obtain, respectively:

$$\frac{\partial u(x_i)}{\partial \tau} = \frac{du(x_i)}{d\tau} - \frac{\partial u(x_i)}{\partial x}\left(1 - \frac{i}{n}\right)\frac{dz}{d\tau} \quad (i = 0, \ldots, n); \tag{19}$$

$$\frac{\partial u(x_i)}{\partial \tau} = \frac{du(x_i)}{d\tau} - \frac{\partial u(x_i)}{\partial x}\cdot\frac{i}{m}\cdot\frac{dz}{d\tau} \quad (i = 0, \ldots, m). \tag{20}$$

After substituting for the partial derivatives $\partial u(x_i)/\partial\tau$ in Eqs. (1)-(6), and introducing finite difference relations of the form $\partial u/\partial x|_{x=x_j} \simeq (u(x_{j+1}, \tau) - u(x_j, \tau))/h$ for Eqs. (1)-(5) and $\partial u/\partial x|_{x=x_j} \simeq (u(x_{j+1}, \tau) - u(x_{j-1}, \tau))/2h$ for Eq. (6), which allows us to take into account the effect of varying $t_s[p(\tau)]$ on the distribution of temperature t_{bw}, we obtain

$$\frac{d\theta_{2,i}}{d\tau}=\frac{(\theta_{2,i}-\theta_{2,i-1})\,m}{z}\left(\frac{i}{m}\cdot\frac{dz}{d\tau}-w_{\mathrm{I}}\right)-\frac{k_1(q)\,\Pi}{(c\gamma s)_a}(\theta_{2,i}-T_{2,i})\quad (i=1,\ldots,m); \tag{21}$$

$$\frac{dT_{2,i}}{d\tau}=\frac{i\,(T_{2,i}-T_{2,i-1})}{z}\cdot\frac{dz}{d\tau}+\frac{k_1(q)}{(c\gamma\delta)_{\mathrm{W}}}(\theta_{2,i}-T_{2,i})-\frac{k_2(q)}{(c\gamma\delta)_{\mathrm{W}}}(T_{2,i}-t_s)\quad (i=0,\ldots,m); \tag{22}$$

$$t_s=t_s\,[p(\tau)]; \tag{23}$$

$$\frac{d\theta_{1,i}}{d\tau}=\frac{n\,(\theta_{1,i}-\theta_{1,i-1})}{L-z}\left[\frac{dz}{d\tau}\left(1-\frac{i}{n}\right)-w_{\mathrm{I}}\right]-\frac{k_1(q)\,\Pi}{(c\gamma s)_a}(\theta_{1,i}-T_{1,i})\quad (i=1,\ldots,n); \tag{24}$$

$$\frac{dT_{1,i}}{d\tau}=\frac{n\,(T_{1,i+1}-T_{1,i})}{L-z}\left(1-\frac{i}{n}\right)\frac{dz}{d\tau}-\frac{k_2(G_{\mathrm{W}})}{(c\gamma s)_{\mathrm{W}}}(T_{1,i}-t_{1,i})+\frac{k_1(q)}{(c\gamma s)_{\mathrm{W}}}(\theta_{1,i}-T_{1,i})\quad (i=0,\ldots,n); \tag{25}$$

$$\frac{dt_{1,i}}{d\tau}=\frac{(t_{1,i+1}-t_{1,i-1})\,n}{2\,(L-z)}\left[\left(1-\frac{i}{n}\right)\frac{dz}{d\tau}+w_{\mathrm{II}}\right]+\frac{k_2(G_{\mathrm{W}})\,\Pi}{(c\gamma s)_{\mathrm{W}}}(T_{1,i}-t_{1,i})\quad (i=1,\ldots,n-1); \tag{26}$$

$$\frac{dz}{d\tau}=\frac{\dfrac{\partial t_s}{\partial p}\cdot\dfrac{dp}{d\tau}-\dfrac{k_2(G_{\mathrm{W}})\,\Pi}{(c\gamma s)_{\mathrm{W}}}\,[T_1(z,\tau)-t_s]}{n\,[t_1(x_1,\tau)-t_s]}\,(L-z)-w_{\mathrm{II}}. \tag{27}$$

The initial and boundary conditions applicable here may be written in the form

$$\theta_{1,n}(z,\tau)=\theta_{2,0}(z,\tau),\quad t_{1,n}(L,\tau)=t_{\mathrm{bW}}(\tau),\quad t_{1,0}(z,\tau)=t_s; \tag{28}$$

$$\theta_{i,j}(0,x)=\theta^0_{i,j}(x),\quad T_{i,j}(0,x)=T^0_{i,j}(x),\quad t_{i,j}(0,x)=t^0_{i,j}(x) \tag{29}$$

$$(i=1,\,2;\ j=0,\ldots,n).$$

In considering the HE separately it is also necessary to assign the perturbations

$$\theta_{2,0}(0,\tau)=\theta_2(\tau),\quad t_{1,n}(L,\tau)=t_{\mathrm{bW}}(\tau),\quad q=f_1(\tau),\quad p=\mathrm{const.} \tag{30}$$

The resulting Eqs. (21)-(30) make it possible to carry out a study of the nonstationary working modes of the HE on a digital computer, by a fourth order Runge-Kutta method, using a standard program.

The initial equations were solved on the Minsk-22 computer with an integration time step of 0.01 sec. The program was written in the language Diana-2 for the linear perturbations

$$\theta_{2,0}=a_1-b_1\tau\geqslant\theta^*_{2,0}, \tag{31}$$

$q = 1-b_2\tau \geq q^*$; $p = a_2ama$ = const; a_i, b_i (i = 1, 2) are constant quantities.

For the initial state we took the data of the stationary state, obtained from the study of the system of Eqs. (21)-(29) indicated above. The results of the computation are shown in Fig. 2.

In making the calculations on the digital computer we subdivided the economizer and evaporator zones into four equal parts; in addition, we considered the equation for the pipeline of boiler water

$$\frac{\partial t_{\mathrm{bW}}}{\partial\tau}+w_{\mathrm{bW}}\frac{\partial t_{\mathrm{bW}}}{\partial x}=0,\quad 0\leqslant x\leqslant l_{\mathrm{p}}; \tag{32}$$

the heat-balance equation for the water volume of the separator, assuming the water level in it to be constant:

$$(c\gamma s)_{\mathrm{S}}H_{\mathrm{W}}\frac{dt_c}{d\tau}=G_{\mathrm{fW}}c_{\mathrm{W}}t_{\mathrm{fW}}+G_{\mathrm{W.\,evap}}i'-G_{\mathrm{W}}c_{\mathrm{W}}t_{\mathrm{S}} \tag{33}$$

and the steam outflow rate, taking the hypothesis of quasistationarity into account:

$$D=\frac{Q}{r(p)}=\frac{\Pi k_2}{r(p)}\int\limits_0^{z(\tau)}(T_2-t_s)\,dx. \tag{34}$$

Results of the HE calculations for the linear perturbations (31) are shown in Fig. 3.

Analyzing the transient processes, there is evidently a tendency towards a "dip" in the steam output with a sharp decline in the input energy.

The problem of how to choose the number of subdivision segments to achieve a given accuracy in the solution depends on the type of the system of equations being solved and the class of functions involved; this problem requires a special study.

NOTATION

q	is the relative (with respect to the nominal volume) rate of flow of heat-transfer agent;
T	is the wall temperature;
t	is the temperature of the working body;
θ	is the temperature of the heat-transfer agent;
c	is the specific heat capacity;
γ	is the density;
g	is the relative rate of vapor flow;
k	is the heat-transfer coefficient;
p	is the pressure;
V	is the volume;
α	is the heat-exchange coefficient;
λ	is the thermal conductivity;
F	is the heat-exchange surface;
s	is the area;
$z(\tau)$	is the variable length of the evaporator section of the heat exchanger;
Π	is the perimeter;
L	is the length of the economizer and evaporator sections;
δ	is the wall thickness;
x	is the space coordinate;
H	is the height of the level in the separator;
G	is the mass flow rate;
w	is the velocity;
τ	is the time;
h	is the integration step;
θ_i, T_i, t_1, t_s $(i = 1, 2)$	are, respectively, the temperature of the alloy, wall, and water in the economizer and evaporator sections, and the temperature of the saturated vapor;
$\theta_2(0, \tau)$	is the inlet disturbance for the alloy;
$\theta_1(L, t)$	is the output temperature of the alloy from the economizer section;
$\underline{G}_{bw}$	is the rate of flow of the boiler water;
$\overline{\theta}_i = \theta_i/\theta_{2n}$;	
$\overline{T}_i = T_i/\theta_{2n}$;	
$\underline{x} = x/4m$;	
$\underline{z}(\tau)$	is the length of the evaporator section;
$\underline{z}$	is the relative length of the evaporator section; $\overline{z} = z/4, 14$ m;
$\underline{\tau}$	is the relative time, $\overline{\tau} = \tau/50$ sec;
$\overline{\theta}_1(L, \tau)$	is the relative output temperature of the alloy from the economizer section, $\overline{\theta}_1(L, \tau) = \theta_1(L, \tau)/269$°C.

Subscripts

n	denotes a nominal value;
fw	denotes feeding water;
bw	denotes boiler water;
S	denotes the separator;
a	denotes heat-transfer agent;
w	denotes water;
W	denotes the wall;
p	denotes the pipeline;
s	denotes a saturation line;
1	denotes the economizer section;
2	denotes the evaporator section;
0	denotes the initial conditions.

A tilde denotes relative values.

LITERATURE CITED

1. I. A. Berezin and N. P. Zhidkov, Computational Methods [in Russian], Vol. 2, Fizmatgiz (1959).
2. M. Kh. Dorri, "A model for the evaporator of a direct-flow steam generator," Izv. Akad. Nauk BSSR, Ser. Fiz.-Tekhn. Nauk, No. 2 (1967).